Additional Chapter 2 review questions

(1) On a particular planet without atmosphere, the height of a certain object in free fall is given by \( h(t) = -5t^2 + 50 \), where \( t \) is time in seconds and height is measured in feet. Find the average velocity of the object over the time interval \([2, 4]\). Find the instantaneous velocity at time \( t \).

(2) Let \( a \) and \( b \) be constants. A function \( f(x) \) is defined by

\[
f(x) = \begin{cases} 
  x^3 & \text{if } x < 2 \\
  a & \text{if } x = 2 \\
  x^2 & \text{if } 2 < x < 3 \\
  b & \text{if } x = 3 \\
  4x - 3 & \text{if } 3 < x
\end{cases}
\]

Find \( \lim_{x \to 2^-} f(x) \), \( \lim_{x \to 2^+} f(x) \), \( \lim_{x \to 3^-} f(x) \), \( \lim_{x \to 3^+} f(x) \). Does \( \lim_{x \to 2} f(x) \) exist? Does \( \lim_{x \to 3} f(x) \) exist? Find \( a \) such that \( f \) is left-continuous at 2. Find \( a \) such that \( f \) is right-continuous at 2. Find \( b \) such that \( f \) is continuous at 3.

(3) Find the vertical asymptotes of the function \( f(x) = \frac{x^2 + x - 2}{x^3 - x} \).

(4) Let \( a \) and \( b \) be constants. A function \( f(x) \) is defined by

\[
f(x) = \begin{cases} 
  x^2 & \text{if } x < 2 \\
  ax + b & \text{if } 2 \leq x \leq 3 \\
  5 - x^2 & \text{if } 3 < x
\end{cases}
\]

Find \( a \) and \( b \) such that \( f \) is continuous.

(5) Prove that the equation \( x = \cos x \) has a solution \( x \) in the interval \((0, \pi/2)\).

(6) Assume that \( f(x) \) is a continuous function defined for all \( x \) in the interval \([2, 4]\). Assume also \( f(2) = 7 \), \( f(3) = 15 \), \( f(4) = 8 \). Prove the existence of two different numbers \( a \) and \( b \) such that \( f(a) = 12 \) and \( f(b) = 12 \).

(7) Prove that the equation \( x^{1/3} - x = 5 \) has a solution \( x \) in the interval \((-1000, 1000)\).

(8) Prove \( \lim_{x \to 2} (3x + 1) = 7 \) using an \( \varepsilon, \delta \) argument.

(9) Prove \( \lim_{x \to 2} (-3x + 7) = 1 \) using an \( \varepsilon, \delta \) argument.

(10) Prove \( \lim_{x \to 0} x^{1/3} = 0 \) using an \( \varepsilon, \delta \) argument.

(11) Find a function \( f(x) \) with domain \([0, 1]\) such that \( f(0) = 2 \), \( f(1) = 4 \), but the equation \( f(x) = 3 \) does not have any solution \( x \).

(12) Find a function \( f(x) \) with domain \((-\infty, \infty)\) such that \( \lim_{x \to 0^-} f(x) \) exists, but \( \lim_{x \to 0^+} f(x) \) does not exist.