Review problems for the Math 151 final exam

(1) Evaluate \( \int_1^2 \frac{\sqrt{x} + 3}{\sqrt{x} + 5} \, dx \).

(2) Evaluate \( \int \frac{dx}{16 + 9x^2} \). Check your answer using differentiation and algebra.

(3) Evaluate \( \int \frac{dx}{\sqrt{16 - 9x^2}} \). Check your answer using differentiation and algebra.

(4) Evaluate \( \int \frac{dx}{x(ln x)(\ln(ln x))^2} \).

(5) Evaluate \( \int \frac{5^x \, dx}{1 + 5^{2x}} \).

(6) Evaluate \( \frac{d}{dx} \int_1^{\sin^{-1} x} \frac{\sin t}{t} \, dt \).

(7) Evaluate \( \int \frac{x^3 \, dx}{\sqrt{1 - 4x^8}} \).

(8) Evaluate \( \int_1^3 |x^2 - 4| \, dx \).

(9) Evaluate \( \int \frac{\sin x \, dx}{|\cos x| \sqrt{25 \cos^2 x - 1}} \).

(10) Evaluate \( \int \frac{dx}{\sqrt{2^{2x} - 1}} \).

(11) Evaluate \( \int x^2 \sqrt{3x + 5} \, dx \).

(12) Evaluate \( \int (\sec x)(\tan x)^{5\sec x} \, dx \).

(13) Evaluate \( \int \tan x \, dx \).

(14) Evaluate \( \int \frac{4x + 5}{x^2 + 9} \, dx \).

(15) Evaluate \( \int (\sec^2 x) \sqrt{\tan x} \, dx \).
(16) Evaluate \( \int \frac{\sec^2(\sqrt{x})}{\sqrt{x}} \, dx \).

(17) Find the area under the graph of \( f(x) = 3x^2 + 2x \) over \([0, 2]\) using \( \lim_{N \to \infty} R_N \). Check your answer using the Fundamental Theorem of Calculus. You will have to use the formulas

\[
\sum_{j=1}^{N} j = \frac{N(N + 1)}{2}, \quad \sum_{j=1}^{N} j^2 = \frac{N(N + 1)(2N + 1)}{6}.
\]

(18) Find the Riemann sum corresponding to the partition \( P : 1 < 3 < 7 < 10 < 15 \), the sample points 2, 4, 9, 12 and the function \( f(x) = x^2 \). What is the norm \(||P||\) of this partition?

(19) Assume that \( f(x) \) is a continuous function on \([1, 10]\) such that

\[
\int_{1}^{8} f(x) \, dx = -5, \quad \int_{3}^{10} f(x) \, dx = 4, \quad \int_{1}^{10} f(x) \, dx = -7.
\]

Find \( \int_{3}^{8} f(x) \, dx \).

(20) Find all functions \( f(x) \) such that \( f''(x) = \cos(2x) \).

(21) An object travels along the \( s \)-axis, where distances are measured in feet. At time \( t \), measured in seconds, the velocity of the object is \( v(t) = t^2 - 4 \) feet per second. Find the total distance traveled by the object during the time interval \([1, 3]\).

(22) Find the area bounded by the curves \( y = x^2, y = 4x - 3 \) using an integral along the \( x \)-axis.

(23) Find the area bounded by the curves \( y = x^2, y = 4x - 3 \) using an integral along the \( y \)-axis. Compare the answer to this problem with the answer to the previous problem.

Since the final exam is cumulative, the next 27 problems are from the first four chapters of the book.

(24) Find the absolute maximum and the absolute minimum of \((2x - 1)^3(3x - 1)^2\) over the interval \([1/3, 1/2]\).

(25) Find the points on the parabola \( x = y^2 \) which are closest to the point \((9, 0)\) in the \( xy \)-plane.
(26) We have a square sheet of cardboard that measures 20 inches per side. We cut out a square measuring $x$ inches per side from each of the four corners of the cardboard. Then we make four folds in the remaining cardboard in such a way that we get an open box with height $x$ inches. What value of $x$ will give us the largest possible volume of this open box?

(27) Consider a function $f(x)$ such that $f'(x) = \frac{x}{(1 + x^4)^2}$. Find the intervals where $f(x)$ is concave up and the intervals where $f(x)$ is concave down. Find the inflection points of $f(x)$.

(28) Explain how Newton’s Method can be used to find a sequence $x_0, x_1, x_2, \ldots$ of successive approximations to a solution of $x^2 = 3$. Find $x_1$ and $x_2$ when $x_0 = 2$.

(29) Find the linearization $L(x)$ of the function $f(x) = x^{1/3}$ centered at $x = 8$.

(30) Consider $f(x) = \frac{x^2}{4 - x^2}$. Find the intervals where $f(x)$ is increasing and the intervals where $f(x)$ is decreasing. Find the intervals where $f(x)$ is concave up and the intervals where $f(x)$ is concave down. Find any relative extrema that may exist.

(31) Consider $f(x) = \frac{x^3}{4 - x^2}$. Find the intervals where $f(x)$ is increasing and the intervals where $f(x)$ is decreasing. Find the intervals where $f(x)$ is concave up and the intervals where $f(x)$ is concave down.

(32) Evaluate $\frac{d}{dx} ((\sec x)^{\tan x})$.

(33) Evaluate $\frac{d}{dx} \left( \sec^{-1}(e^x + 4) \right)$.

(34) Evaluate $\frac{d}{dx} \left( \sin^{-1} \left( \sqrt{x^4 + 2} \right) \right)$.

(35) Evaluate $\frac{d}{dx} \left( \tan^{-1} \left( (x^3 + 1)^4 \right) \right)$.

(36) Evaluate $\frac{d}{dx} \left( \frac{e^{-(x^4+3)}}{\sec(x^2)} \right)$.

(37) Evaluate $\frac{d}{dx} \left( \ln (x^{\cos x}) \right)$.

(38) Find the second derivative of the function $\sec(x^3)$. 

3
(39) A street light is located 20 feet above the sidewalk. A 5 foot high person is walking away from the light at a speed of 3 miles per hour. How fast is the shadow of the person lengthening?

(40) Find the slope of the line tangent to the curve $x^2y^2 + xy = 6$ at the point $(1, 2)$ in the $xy$-plane.

(41) Evaluate $\lim_{x \to \infty} \sqrt[4]{\frac{3x^2 + 5x + 8}{4x^2 - x + 7}}$.

(42) Evaluate $\lim_{x \to 2} \frac{\sqrt{2x + 5} - 3}{\sqrt{3x + 10} - 4}$.

(43) Assume that the function $f(x)$ is defined by

$$f(x) = \begin{cases} 3x^2 + 1 & \text{if } x < 2 \\ a & \text{if } x = 2 \\ 2x^3 + 2 & \text{if } x > 2 \end{cases}$$

where $a$ is a constant. Find $\lim_{x \to 2^+} f(x)$ and $\lim_{x \to 2^-} f(x)$. Prove that there does not exist any choice of $a$ such that the function $f(x)$ is continuous.

(44) Evaluate $\lim_{x \to \infty} \left(1 + \frac{1}{x^2}\right)^{x^2}$.

(45) Evaluate $\lim_{x \to 0} \frac{1 - \cos(5x)}{1 - \cos(7x)}$.

(46) Evaluate $\frac{d}{dx} \left( \frac{1}{x^2} \right)$ using the definition $\frac{d}{dx} (f(x)) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$ of the derivative.

(47) Evaluate $\lim_{x \to \infty} \frac{\sqrt{5x^2 + x + 8}}{x}$ and $\lim_{x \to -\infty} \frac{\sqrt{5x^2 + x + 8}}{x}$.

(48) Simplify $\sec(\sin^{-1} x)$ and $\tan(\sin^{-1} x)$.

(49) Explain why the equation $\ln x = 2 - \frac{x}{e^2}$ must have a solution $x$ in the interval $(e, e^2)$.

(50) Assume $f(x) = \frac{x - 3}{5}$. Find a formula for the inverse function $f^{-1}(x)$. 